MATH 141 HOMEWORK 2 DUE DATE: MONDAY, MAY 15TH

Name: Solutions

This homework is due Monday, May 14th. If you have questions regarding any of this, feel free to ask during office hours or send me an email. When writing solutions, present your answers clearly and neatly, showing only necessary work.

1. Find $\lim_{x\to\pi} \sin\left(\frac{x}{2} + \sin(x)\right)$

$$\lim_{x\to\pi} \sin\left(\frac{x}{2} + \sin(x)\right) = \sin\left(\frac{1}{2}\lim_{x\to\pi} x + \sin\left(\lim_{x\to\pi} x\right)\right)$$
$$= \sin\left(\pi/2 + \sin(\pi)\right) = \sin(\pi/2 + 0) = 1$$

Answer: 1

2. Find

(a) $\lim_{x\to\infty} \sqrt{x^2+1} - 2x$ $= \lim_{x\to\infty} \frac{x^2+1-4x^2}{\sqrt{x^2+1}+2x} = \lim_{x\to\infty} \frac{-3x^2+1}{\left(\frac{\sqrt{x^2+1}}{2x}+1\right)2x}$

$$= \left(\lim_{x\to\infty} \frac{1}{\sqrt{\frac{x^2+1}{4x^2}}+1}\right)\cdot\left(\lim_{x\to\infty} \frac{-3x^2+1}{2x}\right) = \frac{2}{3}\lim_{x\to\infty} \frac{-3x^2+1}{2x} = -\infty$$

Answer: $-\infty$

(b) $\lim_{x\to 0} \frac{\sqrt{x+2}-\sqrt{2-x}}{x}$ $= \lim_{x\to 0} \frac{x+2-(2-x)}{x(\sqrt{x+2}-\sqrt{2-x})} = \lim_{x\to 0} \frac{2}{\sqrt{x+2}-\sqrt{2-x}} = \frac{2}{\sqrt{2}+\sqrt{2}}$

$$= \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

Answer: $\sqrt{2}/2$.

(c) $\lim_{x \to 3} \frac{\sqrt{x^2-5}+2}{x-3} = \lim_{x\to 3} \frac{x^2-5-4}{(x-3)(\sqrt{x^2-5}-2)} = \lim_{x\to 3} \frac{x+3}{\sqrt{x^2-5}-2}$

$\lim_{x\to 3^-}$: $x<3$

$\Rightarrow x^2<9$

$\Rightarrow x^2-5<4$

$\Rightarrow \sqrt{x^2-5}<2$

$\Rightarrow \lim_{x\to 3^-} \frac{x+3}{\sqrt{x^2-5}-2} = -\infty$

$\lim_{x\to 3^+}$: $x>3$

$\Rightarrow x^2>9$

$\Rightarrow x^2-5>4$

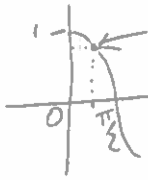
$\Rightarrow \sqrt{x^2-5}>2$

$\Rightarrow \lim_{x\to 3^+} \frac{x+3}{\sqrt{x^2-5}-2} = \infty$

Answer: DNE

3. Use the intermediate value theorem to show, in words, that $f(x) = x + \cos(x)$ has a root in the interval $[-1, 0]$.

$f(-1) = -1 + \cos(-1) = -1 + \cos(1)$.



$\cos(1) \Rightarrow 0<\cos(1)<0 \Rightarrow f(-1)<0$.

$f(0) = 0 + \cos(0) = 1 > 0$.

Since f is continuous and there is a sign chang on $[-1,0]$, by the IVT, $f(x)$ has a root in $[-1,0]$

4. Use the limit definition of the derivative to find $f'(x)$ if $f(x) = \frac{1}{2x+1}$

$$f'(x) = \lim_{h\to 0} \frac{\frac{1}{2(x+h)+1} - \frac{1}{2x+1}}{h} = \lim_{h\to 0} \frac{(2x+1)-(2(x+h)+1)}{h(2(x+h)+1)(2x+1)}$$

$$= \lim_{h\to 0} \frac{-2}{(2(x+h)+1)(2x+1)} = \frac{-2}{(2x+1)^2}$$

Answer: $\frac{-2}{(2x+1)^2}$

5. Find a point on the graph of $f(x) = \frac{x}{2} + \frac{1}{2x+1}$ that has slope $-\frac{3}{2}$

$$f'(x) = \frac{1}{2} + \frac{-2}{(2x+1)^2} = -\frac{3}{2}$$

$$\Rightarrow \frac{2}{(2x+1)^2} = 2$$

$$\Rightarrow (2x+1)^2 = 1$$

$\Rightarrow x = 0$ or -1

Answer: 0 or -1.

The End.